

# Verasco: a Formally Verified C Static Analyzer



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**Joint work with:**

Vincent Laporte, Sandrine Blazy,  
Xavier Leroy, David Pichardie, . . .



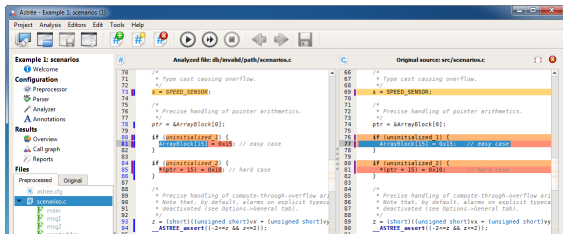
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GdR GPL thesis prize

# Static analyzers

They **automatically** prove the absence of certain kinds of bugs

- Examples : “No invalid memory access”, “No division by 0”, “Small rounding errors”, ...
- Undecidable problem:
  - Success  $\Rightarrow$  no bug of some class
  - In case of doubt, emit an alarm

Exemples : Astrée, Frama-C EVA, Fluctuat, Sparrow, ...



## Abstract interpretation

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**Abstract interpretation:** theory for building static analyzer

- Run the program using an **abstract semantics**
- Always **terminating** computation
- **Soundly** approximating the concrete semantics

**Abstract domains** used to approximate program states

- Ex: variation intervals for integer variables. . .

## Abstract interpretation: example

---

```
x=0
```

```
loop {
```

```
    if x > 11
```

```
        break
```

```
    x+=2
```

```
}
```

## Abstract interpretation: example

```
{T}
x=0
{x = 0}
loop {
  {x ∈ [0, 13] ∧ x mod 2 = 0}
  if x > 11
  {x = 12}
  break
  {x ∈ [0, 11] ∧ x mod 2 = 0}
  x+=2
  {x ∈ [2, 13] ∧ x mod 2 = 0}
}
{x = 12}
```

## Uses of static analyzers

---

Two main use cases:

- Bug finders
  - Very precise, no direct need for correctness
- Program verification
  - Used to prove properties on **critical code**
  - **Need for trust**

Static analyzers are **complex**

- Advanced algorithms (linear optimization, symbolic manipulations, ...)
- Technical problems (floating-point, C semantics, ...)

⇒ **probably buggy**

## Our analyzer: Verasco

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A static analyzer **is a program**, we can **prove it!**

Verasco:

- Programmed and **formally verified** using the Coq proof assistant
- Based on **abstract interpretation**
- Handles **most of C99**
  - industrial, widely used language
  - no dynamic memory allocation, no recursion
- Proves the absence of undefined behaviors
  - dynamic type errors
  - memory errors
  - arithmetic exceptions

Introduction

# Overview of Verasco

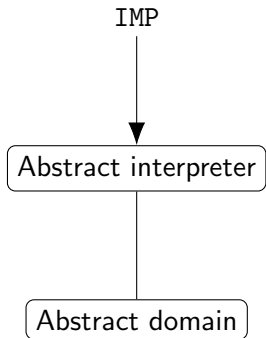
Technical zoom: numerical abstract domains

Conclusions



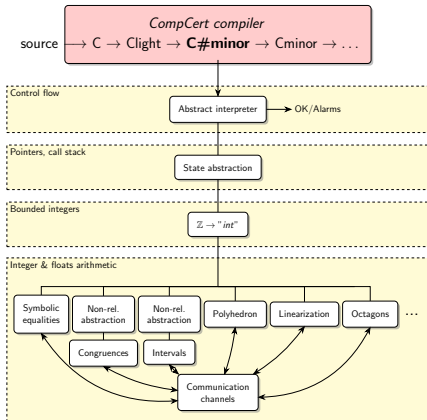
## Textbook formalized static analyzers

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- IMP toy language
- No handling of pointers
- Unbounded integers abstract domain

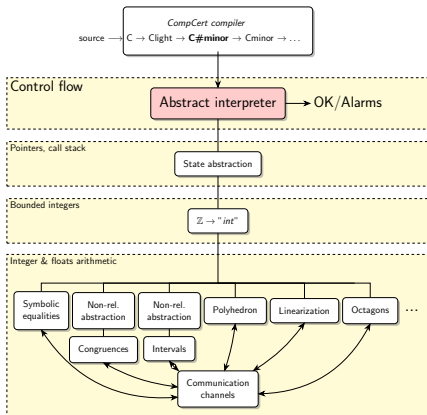
# Modular architecture



## Reuses the **CompCert** front-end

- Until C#minor
- Language simpler than C99
- Uses the same formal semantics as CompCert
  - **Guarantees** provided by the analyzer provably **extend to the assembly code**.
- A priori unsound if used with another C99 semantics

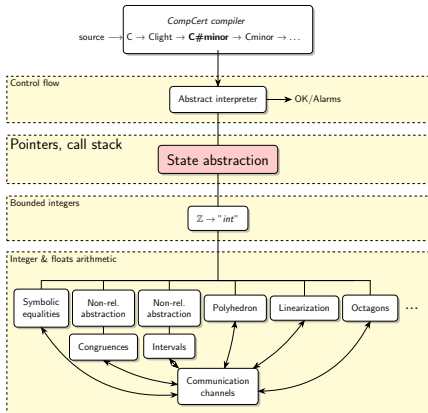
# Modular architecture



## Abstract interpreter

- Fixpoints using widening for loops and gotos
- Proved correct using a Hoare logic for C#minor
- Parameterized by a state abstract domain

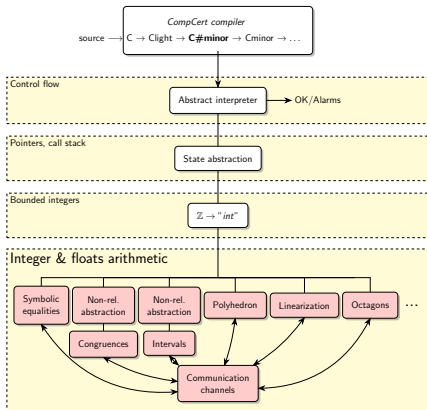
# Modular architecture



## State abstract domain

- Solves pointer references
  - Points-to domain
- Checks type and memory safety
  - Types domain
  - Permissions domain
- Design, implemented and proved correct by V. Laporte
- Parameterized by a numerical abstract domain
  - concretizes to *numerical* environments

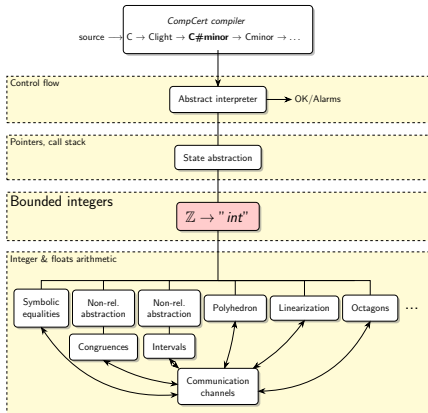
# Modular architecture



## Several numerical domains

- Intervals over  $\mathbb{Z}$  and floats
- Arithmetical congruences
- Symbolic equalities
- Octagons
- Convex polyhedra
  - Contributed by Foulhe, Boulmé and Périn (Verimag)
- Modular communication system using channels

# Modular architecture



## Bounded **machine integers** analysis

- Wraparound when overflow
- Checks numerical errors
  - Division by 0
  - Undefined overflows
- Parameterized by an abstract domain for  $\mathbb{Z}$

## Example 1

---

```
const double sqrt_tab[128] = { ... };

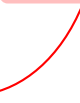
int main() {
    double x = verasco_any_double();
    verasco_assume(0.3 < x && x < 0.7);
    double y = sqrt_tab[ (int)(x*128.) ];
    verasco_assert(0.54 < y && y < 0.84);
    return 0;
}
```

## Example 1

---

```
const double sqrt_tab[128] = { ... };  
  
int main() {  
    double x = verasco_any_double();  
    verasco_assume(0.3 < x && x < 0.7);  
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```

Integer & float  
intervals





# Example 1

State & numerical  
domains cooperation

```
const double sqrt_tab[128] = { ... };  
  
int main() {  
    double x = verasco_any_double();  
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    double y = sqrt_tab[ (int)(x*128.) ],  
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    return 0;  
}
```

Integer & float  
intervals

# Example 1

State & numerical  
domains cooperation

```
const double sqrt_tab[128] = { ... };  
  
int main() {  
    double x = verasco_any_double();  
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    double y = sqrt_tab[ (int)(x*128.) ],  
    verasco_assert(0.54 < y && y < 0.84);  
    return 0;  
}
```

Integer & float  
intervals

Precise bounds on result

## Example 2

---

```
int main() {
    int v0 = 0, v1 = 1;
    int* tab[6] = {&v0, &v1, &v0, &v1, &v0, &v1};

    for(int i = -5; i < 6; i++) {
        int in_range = (0 <= i && i <= 2);
        int some_other_computation = i*i+32;
        if(in_range)
            verasco_assert(*(tab[ 2*i+1 ]) == 1);
    }
    return 0;
}
```

## Example 2

```
int main() {
    int v0 = 0, v1 = 1;
    int* tab[6] = {&v0, &v1, &v0, &v1, &v0, &v1};

    for(int i = -5; i < 6; i++) {
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    }
    return 0;
}
```

Symbolic propagation  
of conditions

## Example 2

```
int main() {  
    int v0 = 0, v1 = 1;  
    int* tab[6] = {&v0, &v1, &v0, &v1, &v0, &v1};  
  
    for(int i = -5; i < 6; i++) {  
        int in_range = (0 <= i && i <= 2);  
        int some_other_computation = i*i+32;  
        if(in_range) {  
            verasco_assert(*(tab[ 2*i+1 ]) == 1);  
        }  
    }  
    return 0;  
}
```

Symbolic propagation  
of conditions

Use of parity  
(congruence domain)

## Example 2

```
int main() {  
    int v0 = 0, v1 = 1;  
    int* tab[6] = {&v0, &v1, &v0, &v1, &v0, &v1};  
  
    for(int i = -5; i < 6; i++) {  
        int in_range = (0 <= i && i <= 2);  
        int some_other_computation = i*i+32;  
        if(in_range) {  
            verasco_assert(*(tab[ 2*i+1 ]) == 1);  
        }  
    }  
    return 0;  
}
```

Symbolic propagation  
of conditions

Complex memory access

Use of parity  
(congruence domain)

## Example 3

---

```
int src[10] = { ... };
int dst[10];

int main() {
    int i_src = 0, i_dst = 9;
    while(i_dst >= 0) {
        dst[i_dst] = src[i_src];
        i_dst--; i_src++;
    }
    verasco_assert(i_src == 10);
    return 0;
}
```

## Example 3

```
int src[10] = { ... };
int dst[10];

int main() {
    int i_src = 0, i_dst = 9;
    while(i_dst >= 0) {
        dst[i_dst] = src[i_src];
        i_dst--; i_src++;
    }
    verasco_assert(i_src == 10);
    return 0;
}
```

Octagons prove the **relational invariant**  
 $i\_src + i\_dst = 9 \dots$



## Example 3

```
int src[10] = { ... };  
int dst[10];
```

```
int main() {  
    int i_src = 0, i_dst = 9;  
    while(i_dst >= 0) {  
        dst[i_dst] = src[i_src];  
        i_dst--; i_src++;  
    }  
    verasco_assert(i_src == 10);  
    return 0;  
}
```

Octagons prove the  
**relational** invariant  
 $i\_src + i\_dst = 9 \dots$

... and deduce precise  
bounds for  $i\_src$

## Modular interfaces between components

---

Operations:

- $\top$     $\sqcup$     $\sqsubseteq$     $\nabla$
- Transfer functions

With Coq specifications

- Concretization function  
 $\gamma : \text{Abs} \rightarrow \text{Concr} \rightarrow \text{Prop}$
- Soundness theorems
- Abstraction function  $\alpha$  is not needed
  - No optimality result

## Example of interface

State abstract domain

---

- Concrete states:



### Operations

- Standard abstract interpretation operators:

- $\sqsubseteq$  :  $\text{abstate} \rightarrow \text{abstate} \rightarrow \text{bool}$
- $\sqcup, \nabla$  :  $\text{abstate} \rightarrow \text{abstate} \rightarrow \text{abstate}$

- Abstract transfer functions:

- `assign, store, assume, push_frame, pop_frame...`

# Example of specification

State abstract domain

---

$$\gamma : \text{abstate} \rightarrow \text{concrete\_state} \rightarrow \text{Prop}$$

## ■ Specifications:

- $\forall a b, \gamma(a) \cup \gamma(b) \subseteq \gamma(a \sqcup b)$
- $\forall x e a, \{\rho[x := v] \mid \rho \in \gamma(a) \wedge \rho \vdash e \Downarrow v\} \subseteq \gamma(\text{assign } x e a)$

## ■ Other domains have similar specifications

## Methodology

---

- Programmed and proved correct in Coq.
- Extracted into OCaml, then compiled into an executable.
- No use of a posteriori validation.
  - Except for the third-party polyhedra domain (Farkas certificates).
- About 45000 lines of Coq.
  - Half proofs, half code & specs

Introduction

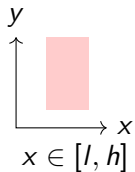
Overview of Verasco

# Technical zoom: numerical abstract domains

Conclusions

# Numerical abstract domains in Verasco

## Intervals



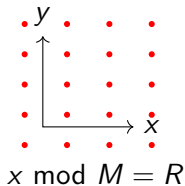
## Symbolic equalities

$$z \doteq y * y$$

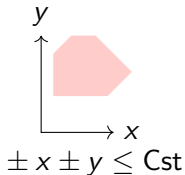
$$c \doteq (x < 1 \ \&\& \ 2 \leq y)$$

$$(y \leq 2.5) \doteq \text{true}$$

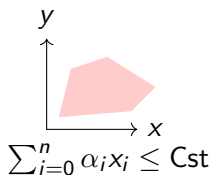
## Congruences



## Octagons



## Polyhedron



# Modularity of numerical abstract domains

---

Modular **communication** system between domain

- **Weakly reduced** product

- **Precise** (vs. direct products) and **practical** (vs. reduced products)
- All abstract domains share a **common interface**
- A domain **pulls** information using **query channels**
- A domain **pushes** information using **message channels**
- Inspired from Astrée

- Easy to soundly add or deactivate an abstract domain

- Flexible precision-performance tradeoff
- Some abstract domains are essential (e.g., intervals) for precision



# Octagons

## Overview

---

- Very popular **weakly relational domain**
  - Originally by Miné from Astrée
- Inequalities of the form:  $\pm x \pm y \leq Cst$
- Interval constraints expressible:

$$x + x \leq 2h \quad -x - x \leq -2l$$

- Data structure: **difference bound matrix**  $A_{xy}$  for  $x, y$  **signed variables**:

$$\forall x, y, x + y \leq A_{xy}$$

# Octagons

## Sparse algorithms

---

Usual algorithms for Octagons:

- Maintain a saturated set of constraints  $\pm x \pm y \leq Cst$
- Problem:

$$\begin{cases} x + x \leq A_{x\bar{x}} \\ y + y \leq A_{y\bar{y}} \end{cases} \implies x + y \leq A_{x\bar{y}} \leq \frac{A_{x\bar{x}} + A_{y\bar{y}}}{2}$$

$\implies$  **Dense constraints** even for interval bounds

Our algorithms for Octagons:

- **Weak** form of saturation
- **Maintain the sparsity** of the constraints

# Octagons

## Example

---

$$\begin{array}{l} +x \\ -x \\ +y \\ -y \\ +z \\ -z \end{array} \begin{pmatrix} -x & +x & -y & +y & -z & +z \\ 0 & & & & & \\ 0 & & & & & \\ 0 & & & & & \\ 0 & & & & & \\ 0 & & & & & \\ 0 & & & & & 0 \end{pmatrix}$$

# Octagons

## Example

Interval constraints:

$$\begin{array}{r}
 +x \\
 -x \\
 +y \\
 -y \\
 +z \\
 -z
 \end{array}
 \begin{pmatrix}
 -x & +x & -y & +y & -z & +z \\
 0 & 2 & & & & \\
 0 & 0 & & & & \\
 & & 0 & 6 & & \\
 & & -2 & 0 & & \\
 & & & & 0 & 4 \\
 & & & & 0 & 0
 \end{pmatrix}$$

$$x \in [0, 1]$$

$$y \in [1, 3]$$

$$z \in [0, 2]$$

# Octagons

## Example

---

Strong saturation  
 $\implies$  Dense matrix

$$\begin{array}{r}
 +x \\
 -x \\
 +y \\
 -y \\
 +z \\
 -z
 \end{array}
 \begin{pmatrix}
 -x & +x & -y & +y & -z & +z \\
 0 & 2 & 0 & 4 & 1 & 3 \\
 0 & 0 & -1 & 3 & 0 & 2 \\
 3 & 4 & 0 & 6 & 3 & 5 \\
 -1 & 0 & -2 & 0 & -1 & 1 \\
 2 & 3 & 1 & 5 & 0 & 4 \\
 0 & 1 & -1 & 3 & 0 & 0
 \end{pmatrix}$$

# Octagons

## Example

---

Weak saturation  
(nothing to do here)

$$\begin{array}{r}
 +x \\
 -x \\
 +y \\
 -y \\
 +z \\
 -z
 \end{array}
 \begin{pmatrix}
 -x & +x & -y & +y & -z & +z \\
 0 & 2 & & & & \\
 0 & 0 & & & & \\
 & & 0 & 6 & & \\
 & & -2 & 0 & & \\
 & & & & 0 & 4 \\
 & & & & 0 & 0
 \end{pmatrix}$$

# Octagons

## Example

---

Adding the constraint:

$$x + y \leq 2$$

$$\begin{array}{r}
 +x \\
 -x \\
 +y \\
 -y \\
 +z \\
 -z
 \end{array}
 \begin{pmatrix}
 -x & +x & -y & +y & -z & +z \\
 0 & 2 & & 2 & & \\
 0 & 0 & & & & \\
 & 2 & 0 & 6 & & \\
 & & -2 & 0 & & \\
 & & & & 0 & 4 \\
 & & & & 0 & 0
 \end{pmatrix}$$

# Octagons

## Example

---

Weak saturation:

$$\begin{array}{r}
 +x \\
 -x \\
 +y \\
 -y \\
 +z \\
 -z
 \end{array}
 \left(
 \begin{array}{cccccc}
 -x & +x & -y & +y & -z & +z \\
 0 & 2 & 0 & 2 & & \\
 0 & 0 & -1 & 2 & & \\
 2 & 2 & 0 & 4 & & \\
 -1 & 0 & -2 & 0 & & \\
 & & & & 0 & 4 \\
 & & & & 0 & 0
 \end{array}
 \right)$$

$$\begin{aligned}
 y + y &\leq 4 \\
 y - x &\leq 2 \\
 -y - x &\leq -1 \\
 x - y &\leq 0
 \end{aligned}$$



# Octagons

## Example

---

Weak saturation:

$$\begin{array}{c}
 +x \\
 -x \\
 +y \\
 -y \\
 +z \\
 -z
 \end{array}
 \left(
 \begin{array}{cccc|cc}
 -x & +x & -y & +y & -z & +z \\
 0 & 2 & 0 & 2 & & \\
 0 & 0 & -1 & 2 & & \\
 2 & 2 & 0 & 4 & & \\
 -1 & 0 & -2 & 0 & & \\
 & & & & 0 & 4 \\
 & & & & 0 & 0
 \end{array}
 \right)$$

$$\begin{aligned}
 y + y &\leq 4 \\
 y - x &\leq 2 \\
 -y - x &\leq -1 \\
 x - y &\leq 0
 \end{aligned}$$

Still sparse

# Abstract domain of symbolic equalities

## Motivating example

---

CompCert front-end transformation:

```
if(0 <= a && a < 10) {  
  ...  
}  
  
⇒  
  
if(0 <= a)  
  tmp = a < 10;  
else  
  tmp = 0;  
if(tmp) {  
  ...  
}
```

Need to reason on the **relation** between a and tmp.

# Abstract domain of symbolic equalities

- Two kinds of equalities:

- $\text{var} \doteq \text{expr}$
- $\text{expr} \doteq \text{bool}$

- Example:

|                               |                                  |   |
|-------------------------------|----------------------------------|---|
| <code>if(0 &lt;= a)</code>    | $(0 \leq a) \doteq \text{true}$  |   |
| <code>tmp = a &lt; 10;</code> | $(0 \leq a) \doteq \text{true}$  | $\text{tmp} \doteq a < 10$                  |
| <code>else</code>             | $(0 \leq a) \doteq \text{false}$ |   |
| <code>tmp = 0;</code>         | $(0 \leq a) \doteq \text{false}$ | $\text{tmp} \doteq 0$                       |
|                               |                                  | $\text{tmp} \doteq (0 \leq a) ? a < 10 : 0$ |
| <br><code>if(tmp) {</code>    | Unfolding tmp                    |   |
| <code>...</code>              | $a \in [0, 9]$                   |   |
| <code>}</code>                |                                  |   |

Introduction

Overview of Verasco

Technical zoom:  
numerical abstract domains

# Conclusions

## Final theorem


---

**Definition** `vanalysis prog = ... iter ...`


**Theorem** `vanalysis_correct:`

$\forall$  prog res tr,  
`vanalysis prog = (res, nil)  $\rightarrow$`   
`program_behaves (semantics prog) (Goes_wrong tr)  $\rightarrow$`   
`False.`

Empty alarm list



No wrong behavior



## Conclusion

---

**Proving** correct a **realistic static analyzer** based on abstract interpretation is **feasible**

- Realistic, feature-rich language (C99)
- Advanced combination of abstract domains

By expliciting the proofs, we **clarified**:

- The specification of abstract domains
- The architecture of a static analyzer
- Their implementation

## Future work

---

- New analysis techniques:
  - Better handling of control flow (trace partitioning, recursion, ...)
  - Other numerical abstract domains (linear filters, arithmetic-geometric progression, pentagons, ...)
  - Support for dynamic memory allocation
- Better performance
  - Faster abstract domains (arrays summarization, variable packing, ...)
  - Better tools for formally verified software
- Using the results of the analysis (Optimizations, other analyzers)
- Experimenting with industrial code

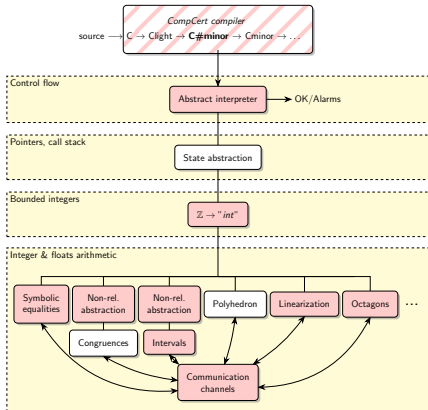
<http://compcert.inria.fr/verasco/>

Questions ?



# Appendices

# My contributions in Verasco



- In CompCert:
  - Parser (verified in Coq)
  - Floats (verified in Coq)
- Abstract interpreter
  - Dedicated program logic
- Most of the numerical domains
  - Handling of machine arithmetic
  - Symbolic equalities
  - Intervals
  - Linearization
  - Octagons
  - Communication channels

## In my thesis...

---

- Design and proof of abstract iterator
  - Handles all C#-minor control constructs
  - Proved using a dedicated program logic
  
- Sharing, hash-consing and memoization in Coq
  - Exemplified on a BDD library
  - Applications in Verasco
  
- Contributions to CompCert
  - Parsing, support for floating-point numbers

## Experiments

---

| Program                  | Size      | Time   |
|--------------------------|-----------|--------|
| <code>integr.c</code>    | 42 lines  | < 0.1s |
| <code>smult.c</code>     | 330 lines | 19.3s  |
| <code>nbody.c</code>     | 179 lines | 10.7s  |
| <code>almabench.c</code> | 352 lines | 5.7s   |

- No alarms on those examples.
  - Pointers, arrays, floats
- Much room for improvement in performance.

# Non-relational numerical abstract domains

## Congruences & Intervals

---

### Intervals

- On  $\mathbb{Z}$  and floating-point numbers.
- Handles all arithmetic and bit-level operations of C99

### Arithmetical congruences

- Needed to check alignment of memory accesses

### Specific interface for non-relational domains

- Common relational adaptation layer

## Complex control flow

---

```
x=0
```

```
loop {
```

```
    if x > 11
```

```
        break
```

```
    x+=2
```

```
}
```

## Complex control flow

```
{T}
x=0
{x = 0}
loop {
  {x ∈ [0, 13] ∧ x mod 2 = 0}
  if x > 11
  {x = 12}
  break
  {x ∈ [0, 11] ∧ x mod 2 = 0}
  x+=2
  {x ∈ [2, 13] ∧ x mod 2 = 0}
}
{x = 12}
```

- Based on **control points**
- We want definitions **following the AST structure**

## Complex control flow

---

x=0

loop {

if x > 11

{x = 12}

break

{⊥, x = 12}

x+=2

}

Dedicated program logic:

$\{P\} s \{Q, Q_b\}$

- Several postconditions
- Defined structurally



## Complex control flow

---

x=0

```
loop {  
  {x ∈ [0, 13] ∧ x mod 2 = 0}  
  if x > 11  
  
    break  
  
  x+=2  
  {x ∈ [2, 13] ∧ x mod 2 = 0, x = 12}  
}
```

Dedicated program logic:

$$\{P\} s \{Q, Q_b\}$$

- Several postconditions
- Defined structurally

## Complex control flow

---

```
x=0
{x = 0}
loop {
    if x > 11
        break
    x+=2
}
{x = 12, ⊥}
```

Dedicated program logic:

$$\{P\} s \{Q, Q_b\}$$

- Several postconditions
- Defined structurally

# Abstract interpreter

Simplified implementation & proof

---

```
Fixpoint iter (ab:abstate) (s:stmt) {struct s}
  : abstate * abstate := ...
```

- Proof step 1: interpreter soundness

```
Lemma iter_ok:  $\forall$  ab_pre stmt ab_post ab_break,
  iter ab_pre stmt = (ab_post, ab_break)  $\rightarrow$ 
  {  $\gamma$ (ab_pre) } stmt {  $\gamma$ (ab_post),  $\gamma$ (ab_break) }.
```

- Proof step 2: program logic soundness

$$\{\cdot\} \cdot \{\cdot, \cdot\} \implies \text{No undefined behavior}$$

# Abstract interpreter

## Big picture

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- Handles all C# minor control constructs
  - Infinite loop, break, if/else, switch, goto, call, return
  - 2 pre-conditions, 4 post-conditions
- Parameterized by a state abstract domain
- Works in a monad for alarms
- Structural recursion on syntax tree
  - Unfolding functions at call sites
- Fixpoint iteration using widening and narrowing
  - One fixpoint iteration per loop
  - Gotos: one fixpoint iteration per function